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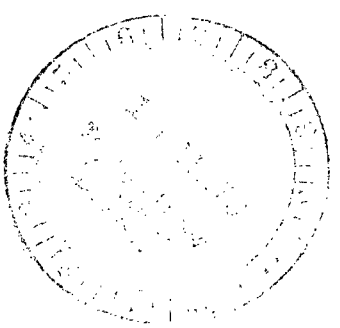
Computation of Atmospheric Attenuation of Sound for Fractional-Octave Bands

Francis J. Montegani

FEBRUARY 1979



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ERRATA

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COMPUTATION OF ATMOSPHERIC ATTENUATION OF SOUND FOR FRACTIONAL-OCTAVE BANDS

Francis J. Montegani
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Page 11: Equation (20) should read

$$S = g(0) + \sum_{j=1}^{m} [g(-j) + g(+j)]$$

Page 20, line 16: The second computer program variable should be CR instead of CM.

Page 22, Fortran statement 4: The constant 4.0 in two places should be 10.0.

Page 23, Fortran statement 7: The constant 7.0 in two places should be 10.0.

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Computation of Atmospheric Attenuation of Sound for Fractional-Octave Bands

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NASA
National Aeronautics
and Space Administration

**Scientific and Technical
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1979

SUMMARY

The atmosphere is acoustically dissipative and causes significant attenuation of propagating sound in addition to the attenuation caused by spherical spreading. Since the effect is frequency dependent, resulting in observed spectra that are not representative of the source, it is important to account for it in much acoustical work. Commonly used acoustical data are the results of fractional-octave band analyses of spectra, whereas atmospheric attenuation is known fundamentally for discrete-frequency sounds. Correct methods of accounting for atmospheric attenuation in band data require consideration of the integrated effect across the bands for the specific distance involved. Such methods have not found their way into general use. Published analyses and computer programs have problems that inhibit implementation of correct methods by potential users. This report illustrates the underlying simplicity of an integral approach, documents the basic analysis, and gives examples of errors incurred by not using correct methods. Computer programs are provided that are understandable, efficient, and simple to use. It is hoped that this will facilitate more widespread use of correct computational methods, especially where routine computer processing of data is employed.

INTRODUCTION

Spectrum analysis of air-propagated sound is fundamental to experimental acoustics. Most spectrum analysis data, particularly where they are associated with human annoyance problems as in aircraft noise certification or in the general area of noise reduction research, are comprised of sound pressure levels in 1/3-octave frequency bands. These wide-band data are commonly used as an alternative to narrow-band or power spectral density data to allow frequency-dependent calculations to be made with a reasonably manageable amount of data.

It is well known that the atmosphere is acoustically dissipative and causes significant attenuation of propagating sound in addition to the attenuation caused by spherical spreading. The effect varies with frequency so that, as propagation distance increases, the observed spectrum differs more and more from that emitted by the source. Thus, a pervasive problem in noise analysis is to account for atmospheric attenuation - either to remove the effect in order to obtain so-called lossless data, which reveal the true character of a source, or to incorporate it into lossless data to make practical propagation calculations. Often it is necessary to adjust measured data to standard conditions,

since atmospheric attenuation is variable with ambient conditions. This requires two steps: removal of the effect due to the measurement conditions, and then reincorporation of it for standard conditions.

Atmospheric absorption is defined fundamentally for discrete-frequency sounds, and is usually expressed as an attenuation per unit distance. Analytically it can be specified as a continuous function of frequency and atmospheric variables. The nature of the phenomenon is well understood, and there is a body of literature on the subject. For this report, the physics of the phenomenon is not an issue. It is presumed, simply, that a suitable analytical representation of discrete-frequency atmospheric attenuation is available in the form of a continuous function of frequency.

When the detailed spectrum of a sound is known, as in having the power spectral density function or the results of narrow-band analysis, accounting for the effect of atmospheric absorption is trivial in principle. The appropriate calculations are made either functionally or discretely over the frequency range. In practice this is rarely, if ever, done.

For the more commonly used data obtained from wide-band analysis of spectra, however, the problem is less simple. Since a band sound pressure level is by definition a measure of the integral of the power spectral density between the lower and upper band-edge frequencies, a single-valued measure of atmospheric absorption for a band can only be obtained by considering the integrated effect over the band. Occasionally this fact is not recognized. Most often, however, it is, but for the sake of expedience the rigorous calculation called for is deliberately avoided. Instead, the band attenuation is taken as that of some discrete frequency, the value of which is determined by some rule of thumb.

The preeminent example of this approach is the widely used procedure specified in ARP-866A (ref. 1). Attenuation for a frequency band is normally obtained by using the discrete frequency attenuation at the band center frequency. For cases where the slope of the spectrum is suitably conditioned and at low frequency where the slope of the attenuation function is small, this is a reasonable approximation. Unfortunately, these conditions do not always prevail, especially at high frequency. This is partially accounted for in ARP-866A by the requirement that the lower band-edge frequency should be used instead of the center frequency when the latter exceeds a certain value. But the procedure still is approximate and requires suitably conditioned spectra if errors are to be small. The ARP-866A procedure also results in band atmospheric attenuations that are linearly proportional to distance. It will be shown later that this also is an approximation.

A correct computational method to obtain single-valued band attenuations requires consideration of integrals across the bands. However, such correct methods have not found their way into common use, and approximate methods prevail. This is true even where routine computer processing of data is employed, which seems to be unjustifiable.

Errors of varying magnitudes are introduced that depend on the shapes of the spectra involved, the absolute value of the frequencies, and the propagation distances. Examples of these errors are given later.

Recent publications (refs. 2 and 3) deal with the correct computation of band attenuations. However, in the author's view, the analyses given therein are neither as succinct nor as understandable as they might be. Furthermore, the computer programs that result are lengthy and complicated, and the methods have other problems that are discussed later. These conditions would seem to keep potential users from implementing correct methods.

The intent of the present work is to document the underlying analysis in a straightforward way and to develop a method incorporated in computer programs that are simple, understandable, and easily employed. This is done in the hope that correct methods will find wider routine use, especially where computer processing of data is employed.

The theoretical development given herein is rigorous and makes use of the relationships, which are a matter of definition, between the power spectral density of a sound signal and wide-band analysis representations of it. Since the resulting method is intended for computing band atmospheric attenuation when only band levels are known, the spectrum shape within each band must be inferred from the band data. This is the only substantive approximation required. However, rather than being a weakness, this approximation is a key element that distinguishes the integral approach from the discrete-frequency method. Any alternative scheme for inferring the spectrum shape may be substituted at the discretion of the reader. The one given herein, which uses straight-line segments, has been used satisfactorily by the author since 1968.

Computer programs, one for performing the band integrations and another for generating discrete-frequency atmospheric attenuation, have been published previously (ref. 4). However, a detailed analysis of the procedure was not given, and, in fact, the band integration program contained some shortcomings. These included a lack of distinction between lossless or air-attenuated input data, a linear dependence of band attenuation on distance, and a minor frequency-stepping error across each band. These are all removed here.

The discrete-frequency attenuation program of reference 4 was based on the graphical information given in reference 5. It should be noted that the specific discrete-frequency attenuation model employed is not critical to the development of the band integration procedure, and any model may be substituted. In reference 2, an improved discrete-frequency attenuation model is presented. This model has been adopted for the present work on the basis of its merits, particularly the wide frequency range of its applicability.

The band attenuation computational procedure developed in reference 2 also has some problems. The most significant is that the number of bands diminishes by two each time an atmospheric attenuation calculation is made. The first and last bands are

lost because of the computational technique employed. Thus, if air-attenuated data are to be converted to standard-day data through the intermediate step of obtaining lossless data, which is a common requirement, a total of four bands will be lost.

The method developed herein eliminates this shortcoming. In addition, the present development is structured with a clear separation between the numerical integration procedure and the technique to infer the spectrum shape from the band data. This makes it easy for the reader to modify or replace the technique used to infer the spectrum shape. Furthermore, the formulations are developed to facilitate computer coding, and frequency incrementing is accomplished by methods that generate so-called preferred frequencies.

Finally, the development given herein defines a band as that portion of a spectrum between certain frequency limits. This is equivalent to specifying measured data obtained with ideal filters. In reality, band data are obtained experimentally with bandpass filters. It has been shown in reference 2 that for extreme spectrum or attenuation gradients the filter skirt can have a significant effect on a spectrum analysis. Such extreme conditions are not usual, however, and to account for the effect of the filter skirt greatly complicates the development. The attendant discussion would need to go far beyond the scope of the present work. For the usual frequency range from 50 to 20 000 hertz, and even to higher frequencies as will be seen, the filter skirt is not likely to present a problem. For further discussion of this point, and for quantitative estimates of the conditions under which filter characteristics can present problems, the reader is referred to reference 2.

SYMBOLS

$A_i(r)$	band atmospheric attenuation, dB
a(f)	discrete-frequency atmospheric attenuation per unit distance, dB/m
b	fractional-octave band size designation denoting 1/b-octave bands
C	constant
f	frequency, Hz
f_c	band center frequency, Hz
f_1, f_2	lower and upper band-edge frequencies, respectively, Hz
g	function
i	band index
j	subband index

k	incremental propagation path index
$L_i(r)$	lossless fractional-octave band sound pressure level, dB
$L_i'(r)$	air-attenuated fractional-octave band sound pressure level, dB
$L_j(r)$	to eq. (14), lossless subband sound pressure level; for eq. (21), lossless or air-attenuated subband sound pressure level, dB
$L_j'(r)$	air-attenuated subband sound pressure level, dB
ΔL_j	subband sound pressure level relative to subband level at band center frequency, dB
m	integer
N_b	band number
n	number of subbands in fractional-octave band
$p_j(r)$	subband rms acoustic pressure, Pa
r	propagation path length, m
S	sum
$W(f, r)$	power spectral density of lossless acoustic pressure, Pa^2/Hz
$W'(f, r)$	power spectral density of air-attenuated acoustic pressure, Pa^2/Hz

ANALYSIS

Let $W(f, r)$ denote the power spectral density of the rms acoustic pressure of freely propagating sound at any distance r from a source in the absence of atmospheric attenuation. This represents the source (lossless) spectrum. If $a(f)$ denotes the discrete-frequency atmospheric attenuation in units of decibels per unit distance, the air-attenuated spectrum $W'(f, r)$ in a homogeneous atmosphere is expressed as

$$W'(f, r) = W(f, r)10^{\frac{-a(f)r}{10}} \tag{1}$$

(It is understood that atmospheric attenuation is dependent on ambient temperature and humidity. These variables are omitted from the functional notation $a(f)$ for simplicity.)

The sound pressure level $L_i(r)$ of the portion of a spectrum in the i^{th} band between the frequency limits of $(f_1)_i$ and $(f_2)_i$ is expressed in terms of the power spectral

density as

$$L_i(r) = 10 \log \int_{(f_1)_i}^{(f_2)_i} W(f, r)df + C \qquad (2)$$

where C is an arbitrary constant required by the definition of a level.[1] The air attenuation in decibels $A_i(r)$ for a spectrum band is the difference between the band level of the lossless spectrum at some location r and the corresponding band level of the air-attenuated spectrum:

$$A_i(r) = 10 \log \int_{(f_1)_i}^{(f_2)_i} W(f, r)df - 10 \log \int_{(f_1)_i}^{(f_2)_i} W'(f, r)df \qquad (3)$$

If the air-attenuated spectrum in the second term on the right of equation (3) is replaced by equation (1), equation (3) becomes

$$A_i(r) = 10 \log \int_{(f_1)_i}^{(f_2)_i} W(f, r)df - 10 \log \int_{(f_1)_i}^{(f_2)_i} W(f, r)10^{\frac{-a(f)r}{10}} \qquad (4)$$

Equation (4) expresses the band attenuation in decibels in terms of the lossless spectrum.

It is just as likely that the air-attenuated spectrum and not the source spectrum will be known. It will be necessary to compute the band attenuation from these data also. To develop a suitable expression for this case, equation (1) is rewritten as

$$W(f, r) = W'(f, r)10^{\frac{+a(f)r}{10}} \qquad (5)$$

[1] As noted in the INTRODUCTION, this formulation is equivalent to specifying a measurement using an ideal filter. If a filter transmission characteristic t(f) is to be accounted for, where $0 \leq t(f) \leq 1$, the function t(f) must be introduced as a factor into the integrand and the limits taken from 0 to ∞. Subsequent development would proceed accordingly.

and substituted in the first term on the right of equation (3) to obtain

$$A_i(r) = 10 \log \int_{(f_1)_i}^{(f_2)_i} W'(f, r) 10^{\frac{+a(f)r}{10}} df - 10 \log \int_{(f_1)_i}^{(f_2)_i} W'(f, r) df \qquad (6)$$

Equation (6) is equivalent to equation (4) but is expressed in terms of the air-attenuated spectrum.

Note that equations (4) and (6) both contain the propagation distance r within an integrand from which it cannot be removed. This establishes a nonlinear dependence of band attenuation on distance, which discrete-frequency approximate methods inherently neglect.

It is also appropriate to note here that the foregoing analysis can be simply extended to an inhomogeneous atmosphere. The quantity $a(f)r$ expresses the total discrete-frequency attenuation in decibels along the propagation path. To extend the results for an inhomogeneous atmosphere it is only necessary to define the total discrete-frequency attenuation along the path as

$$\int_r a(f) dr$$

where $a(f)$ is an implicit function of r by virtue of its dependence on atmospheric variables. Such an extension for making practical calculations is considered later.

If the symbols $L_i(r)$ and $L_i'(r)$ are used in place of the integral expressions for the band levels in equation (3), simple rearrangement results in

$$L_i'(r) = L_i(r) - A_i(r) \qquad (7)$$

and

$$L_i(r) = L_i'(r) + A_i(r) \qquad (8)$$

Lossless, freely propagating sound obeys the inverse square law so that, for two distances r_1 and r_2 along the path of propagation, the lossless spectrum band levels are related by

$$L_i(r_2) = L_i(r_1) + 20 \log\left(\frac{r_1}{r_2}\right) \qquad (9)$$

If r_2 replaces r in equation (7) and equation (9) replaces the first term on the right, equation (7) becomes

$$L_i'(r_2) = L_i(r_1) + 20 \log\left(\frac{r_1}{r_2}\right) - A_i(r_2) \tag{10}$$

Also, substituting r_1 for r in equation (8) and replacing the first term on the right of equation (9) with the result yield

$$L_i(r_2) = L_i'(r_1) + 20 \log\left(\frac{r_1}{r_2}\right) + A_i(r_1) \tag{11}$$

Equations (10) and (11), in conjunction with equations (4) and (6), respectively, provide the means to account for atmospheric attenuation in spectrum analysis band data. Equations (4) and (10) permit determination of the air-attenuated band levels at any distance when the lossless levels at some distance are known. Equations (6) and (11) permit determination of the lossless band levels at any distance when the air-attenuated levels at some distance are known.

NUMERICAL APPROXIMATION

Equations (4) and (6) contain integral expressions involving power spectral density. It is obviously easy to obtain equivalent finite-sum expressions that can be evaluated as precisely as required. However, in most practical computational problems, only band sound pressure level data are available - not power spectral density, which the expressions apparently require. Fortunately, there is a natural connection between power spectral density and the spectral increments involved in finite sum expressions, which permits the power spectral density to be replaced by sound pressure levels in finite frequency subintervals within a band. These sound pressure levels can be obtained by inference from the band data, and essentially reflect the spectrum shape. In this section, numerical integration expressions for band attenuation and related equations for frequency incrementing are developed, and the scheme to infer the spectrum shape from the data is described.

Band Attenuation

The integral expressions for band levels can be approximated directly by finite sums. Equation (2), for example, can be approximated by

$$L_i(r) = 10 \log \sum_{j=1}^{n} W(f_j, r)(\Delta f)_j + C \tag{12}$$

where the i^{th} frequency band is divided into n contiguous intervals, each embracing f_j and each of width $(\Delta f)_j$. (Note that for notation simplicity the subscript i is omitted from the frequency quantities on the right since their restriction to the i^{th} band is clear.) The representation may be made as accurate as necessary by increasing the value of n and choosing f_j and the intervals $(\Delta f)_j$ in such a way that $W(f_j, r)$ approximates the mean value on each interval.

By definition, the power spectral density is related to the rms acoustic pressure $p_j(r)$ in a band embracing f_j by

$$W(f_j, r) = \left. \frac{p_j^2(r)}{(\Delta f)_j} \right|_{(\Delta f)_j \to 0}$$

Thus, equation (12) may be written

$$L_i(r) = 10 \log \sum_{j=1}^{n} p_j^2(r) + C \tag{13}$$

where the $p_j(r)$ $(j = 1, 2, \ldots, n)$ denote the rms acoustic pressures in sufficiently small intervals that collectively comprise the i^{th} band.

From the foregoing and the fact that the rms sound pressure is related to the sound pressure level $L_j(r)$ in decibels (which is the customary form for acoustical data) by

$$p_j^2(r) = p_{ref}^2 10^{\frac{L_j(r)}{10}}$$

equations (4) and (6) can be written as

$$A_i(r) = 10 \log \sum_{j=1}^{n} 10^{\frac{L_j(r)}{10}} - 10 \log \sum_{j=1}^{n} 10^{\frac{L_j(r)-a(f_j)r}{10}} \tag{14}$$

and

$$A_i(r) = 10 \log \sum_{j=1}^{n} 10^{\frac{L'_j(r)+a(f_j)r}{10}} - 10 \log \sum_{j=1}^{n} 10^{\frac{L'_j(r)}{10}} \tag{15}$$

Equations (14) and (15) express atmospheric attenuation, in decibels, for bands of frequency as functions of sound pressure levels in smaller intervals within the bands. To carry out computations it is only necessary to choose frequency intervals across the band and to determine the sound pressure levels in the intervals.

Frequency Intervals

For fractional-octave band analysis, the bandwidth increases in proportion to frequency and the data are conveniently exhibited on a logarithmic frequency scale. It follows that, for summation purposes in equations (14) and (15), sound pressures are more appropriately specified at equal increments of the logarithm of frequency.

Delineation of a fractional-octave band on a logarithmic frequency scale is shown schematically in figure 1. The band center frequency f_c is shown by the bold dashed line and the center frequencies of consecutive bands would appear at equal spacing. The frequencies of the lower and upper band-edges are shown by the bold solid lines. The center frequency of a band is the geometric mean of the lower and upper band-edge frequencies (i. e., $f_c = \sqrt{f_1 f_2}$), and on a logarithmic scale it appears at the center of the band.

The band is shown subdivided by light solid lines into n frequency intervals consistent with the requirements for the summations of equations (14) and (15). Hereinafter these intervals are referred to as subbands. The center frequency of each subband is denoted by the short dashed lines. As will be seen, it is convenient to use an

odd number of subbands; therefore, n is defined according to

$$n = 2m + 1 \tag{16}$$

where m is some integer. The subbands can now be indexed on j from -m to m, as shown in the figure, resulting in the subscript j=0 for the center subband.

If the notation 1/b-octave is used to designate the size of fractional-octave bands, where b is some number (usually, but not necessarily, an integer), the center frequencies of successive bands are obtained from

$$(f_c)_i = 10^{\frac{3}{10b} i} \tag{17}$$

as i takes successive integer values. (This formula restricts frequencies to standard preferred values as specified in ref. 6.) Referring to figure 1, the reader can verify that the center frequency of the j^{th} subband in the i^{th} band is given by

$$(f_j)_i = (f_c)_i 10^{\frac{3}{10b}\left(\frac{j}{2m+1}\right)} \tag{18}$$

Equation (18) permits regular indexing of frequency across the band for use with equations (14) and (15) if the summation limits go from -m to m.

Programming Considerations

Equations (14) and (15) with indexes revised according to the discussion of the preceding section each involve summations S having a functional form

$$S = \sum_{j=-m}^{m} g(j) \tag{19}$$

This can be written equivalently as

$$S = g(0) + \sum_{j=1}^{m} \left[g(-j) + g(+j) \right] \tag{20}$$

Although the form of equation (20) is less simple than that of equation (19), equation (20) is more suited to programmed computation. For either form, an initial value for the accumulating sum must be set. However, equation (20) allows the initialization step to be used to compute g(0). The summation of equation (20) then proceeds from 1 to m, which requires only half the number of cycles than equation (19) requires. The subscript j=0 denotes the band center frequency; hereinafter the 0 is replaced with c.

If the foregoing considerations are applied to equations (14) and (15), the function of r notation is dropped for simplicity, and some rearrangement is made, these equations can be combined into

$$A_i = \pm 10 \log \left\{ \frac{10^{\frac{L_c \pm a(f_c)r}{10}} + \sum_{j=1}^{m} \left[10^{\frac{L_{-j} \pm a(f_{-j})r}{10}} + 10^{\frac{L_j \pm a(f_j)r}{10}} \right]}{10^{\frac{L_c}{10}} + \sum_{j=1}^{m} \left(10^{\frac{L_{-j}}{10}} + 10^{\frac{L_j}{10}} \right)} \right\} \tag{21}$$

where L is now used to denote either lossless or air-attenuated subband levels. The plus signs are used with air-attenuated levels and the minus signs are used with lossless levels.

Note that a constant added to all levels in equation (21) has no effect on the result since it introduces a common factor into all terms of both numerator and denominator. This permits equation (21) to be further simplified by expressing all levels relative to the level at the band center frequency. Equation (21) now can be written in final working form as

$$A_i = \pm 10 \log \left\{ \frac{10^{\frac{\pm a(f_c)r}{10}} + \sum_{j=1}^{m} \left[10^{\frac{\Delta L_{-j} \pm a(f_{-j})r}{10}} + 10^{\frac{\Delta L_j \pm a(f_j)r}{10}} \right]}{1 + \sum_{j=1}^{m} \left(10^{\frac{\Delta L_{-j}}{10}} + 10^{\frac{\Delta L_j}{10}} \right)} \right\} \tag{22}$$

where ΔL is the subband level relative to the subband level at the band center frequency. Values of frequency for use in equation (22) are obtained from equation (18).

It was noted earlier that extension of the analysis to an inhomogeneous atmosphere involved only the total discrete-frequency attenuation along the propagation path. This is the quantity $a(f)r$, which has retained its identity throughout. To use equation (22) for the inhomogeneous case, it is only necessary to replace the total attenuation along the path with

$$\sum_k a_k(f)(\Delta r)_k$$

where the subscript k denotes subdivision of the path into small increments Δr over which $a(f)$ is constant, and for which

$$\sum_k (\Delta r)_k = r$$

Such an extension is well suited to a layered atmosphere model.

Subband Sound Pressure Levels

It is not possible to obtain the rms pressure distribution across a frequency band from band data alone, since each band level inherently represents the integration of the distribution. However, the distribution can be approximated.

It can be shown that a power spectral density function that varies as a constant power of frequency (i. e., the power spectral density level exhibits a straight line relationship on a log frequency scale) results in fractional-octave band levels that likewise vary as a constant (but different) power of band center frequency, irrespective of the size of the bands. This establishes that, for fractional-octave band spectrum analyses presented graphically as band pressure level against logarithm of frequency, the slope of the line connecting the band levels at their respective center frequencies is independent of the size of the bands. The absolute levels obtained from analyses using different bandwidths, however, differ by ten times the logarithm of the bandwidth ratio. These properties are illustrated in figure 2. Thus, subject to an assumption about the smoothness of the spectrum, it is possible to infer from a fractional-octave band spectrum analysis the analysis in yet smaller fractional-octave bands. The number of subbands into which a band may be divided may be chosen without limit.

On the basis of the foregoing considerations, a scheme has been developed to approximate the spectral distribution across fractional-octave bands so that band atmospheric attenuation can be computed with the equations developed. This is illustrated in figure 3. Conceptually, for a given spectrum analysis, the levels of the fractional-octave bands, plotted at the band center frequencies on a logarithmic frequency scale, are connected by straight-line segments. The distribution (but not the absolute values) of the subband levels across each band from edge to edge is thus established approximately by a pair of straight-line half-segments whose slopes are determined by the data. For the first and last bands, where only one of the half-segments is defined by the data, the slope is assumed the same on both sides of the band.

It was shown in the previous section that only the relative distribution of the subband levels across the band was significant. Equation (22) therefore was written in terms of levels relative to the level at the band center frequency. Definition of the relative subband levels using straight-line half-segments is illustrated in figure 4. The fact that only the relative distribution of the subband levels is significant restricts attention to the level increment from band to band; this is loosely referred to as the slope of the data.

Finally, it should be obvious that when discrete tones are present in a spectrum in small numbers to the extent that one, or at most a few, influences the level of any particular band the procedure given here does not yield an accurate subband level distribution. In such a situation, where a tone-dominated band stands out and where for lack of information about the tone frequencies the procedure of this section is used straightforwardly, the computed band attenuation value will be strongly influenced by the discrete tone value at the band center frequency. The error introduced is potentially of no more seriousness than neglecting the integral approach altogether. On the other hand, if the discrete tone frequencies (and their relative levels, if there are more than one within a given band) are known, accurate calculations can be made. In this case, the frequencies used for the summation of equation (22) would be obtained not by equation (18) but by the tone frequencies known to exist, and the summation would proceed only for as many terms as there are discrete tones, possibly only one. If this were done, care would also be needed to remove the effect of the tones before establishing the line segments, and ultimately the attenuation, for the bands adjacent to the tone-dominated band.

COMPUTER PROGRAMS

Equations (10), (11), (17), (18), (22), and a description of atmospheric absorption as a function of frequency constitute a rigorous numerical method to calculate atmospheric attenuation in spectrum bands. The accuracy is limited only by the number of terms in

the summations and by the scheme used to infer the subband levels from the band data. The scheme used herein was described in the preceding section and is illustrated in figure 4.

A pair of Fortran computer programs that embodies this method is given in the appendix. The first program, BASPAT, which obtains the subband levels and performs the numerical integration, has been designed for efficiency, computational power, and simplicity of use. Input data comprise sound pressure levels in contiguous fractional-octave bands at some specified distance. The size of the bands is selectable.

If the input data are lossless, air-attenuated data will be generated. If the input data are air-attenuated, lossless data will be generated. The distance for which the output data are obtained may be different than the distance for the input data. To adjust measured air-attenuated data to standard-day air-attenuated data, two calls of the program are necessary. The first call will generate lossless data, which must then be used in a second call to obtain standard-day data.

The values of all frequencies used in the program are obtained by methods that generate preferred numbers consistent with international standards. For further information on these standards, the reader is referred to references 6 and 7. Briefly, the center frequency of any band can be expressed as

$$f_c = 10^{N_b/10} \tag{23}$$

where N_b is defined as the band number. The band number, moreover, must conform to

$$N_b = \frac{3}{b} n \tag{24}$$

where b denotes the kind of band as defined in the Frequency Intervals section and n is some integer. (Note that the band number N_b need not be an integer.)

In using the program, the user specifies the values of the band center frequencies corresponding to the data simply by specifying a single variable. This is the international band number of the first band, denoted IBNF in the program. The user must insure that IBNF is in conformance with equations (23) and (24). The program uses this value to compute the center frequency of the first band and all succeeding bands of the air attenuated data.

An alternative version of BASPAT is also given in the appendix. This is BASLAT and is functionally the same as BASPAT except that it allows calculations for a layered atmosphere in accordance with previous discussion. It is provided as a convenience to the reader, and all discussions herein with regard to BASPAT apply to it as well.

The program BASPAT calls a subroutine to obtain discrete-frequency atmospheric attenuation. The second program, ATMAT, is provided for such calls. The program ATMAT computes discrete-frequency atmospheric attenuation in accordance with the data presented in reference 2. Obviously, the reader can substitute his own version of this subroutine.

SAMPLE CALCULATIONS

In this section, results obtained by the method and computer programs given herein are obtained for comparison with results from the numerical integration method of reference 2. In the subsequent discussion, examples of errors caused by discrete-frequency approximations of numerical integration methods are presented.

To illustrate the use of the programs, and for convenient evaluation of the results, the sample problem of reference 2 is repeated here. The original data for starting the calculations appear in table I, column 3. The data are presumed to be lossless sound pressure levels 20 meters from the source. The first calculation gets the corresponding air-attenuated data at the same distance for an ambient temperature of 293 K and a relative humidity of 50 percent. To perform the calculation, the subroutine BASPAT is executed from a suitable calling program in which the input variables in the subroutine vector are set as follows:

SLI array of original lossless 1/3-octave band sound pressure levels as shown in table I, column 3

RI 20.0, propagation distance from the source in meters associated with SLI values

IBNF 36, which, in accordance with eqs. (23) and (24) for b = 3, specifies that the frequency associated with the first band level is 4000 Hz nominal (see table I, columns 1 and 2)

NF 15, number of bands

NB 3, designates 1/3-octave bands

T 293.0, ambient temperature, K

RH 50.0, relative humidity, percent

IDK 1, declares input data are lossless, which will generate air-attenuated results

RO 20.0, distance from the source in meters for which the results are desired

The results of this calculation by the program BASPAT are shown in table I, column 4. The corresponding results from reference 2 appear in column 5. As in reference 2, these results (column 4) are next used as data for the inverse calculation.

For this case, the variable IDK in the BASPAT calling vector is set to zero to declare that the input data are air attenuated. All other data remain the same. The results of the inverse calculation appear in column 6. The corresponding results from reference 2, computed from column 5, appear in column 7.

DISCUSSION

Table I shows that the air-attenuated results from the two methods agree exactly at 6300 hertz. Below 6300 hertz, because of the reduced gradient of the discrete-frequency air-attenuation function with frequency, exact agreement prevails. Above 6300 hertz, the results differ by a slightly increasing amount, never exceeding 0.7 decibel up to 80 000 hertz in spite of absolute adjustments to the original data of close to 55 decibels. The differences are attributed to neglecting the filter characteristic in the present procedure. The filter characteristic is of significance where the gradient of the product of air absorption and power spectral density is high, which generally occurs at high frequency. The gradually increasing effect is due to the significantly increasing slope of the air attenuation. The filtered values, column 5, are higher because of the filter skirts, which pass energy from adjacent bands. Nevertheless, neglecting the skirt results in fairly small errors, particularly in consideration of the extremely high frequencies of the example. The present method also results in no loss of data, which is not true of the method of reference 2.

The inverse results obtained by the present method agree to one decimal place except at the highest frequency. A corresponding evaluation of the method of reference 2 cannot be made because of the units precision reported for the lossless results (column 7) and the loss of band data.

With regard to obtaining lossless data from air-attenuated data accounting for the filter characteristic, it must be pointed out that reference 2 contains an implicit error. The development given therein postulates that the air-attenuated (received) spectrum is known. This spectrum corresponds to air-attenuated band data obtained from ideal filters. However, the practical situation for which the equation is developed provides air-attenuated data obtained from real filters. Thus the resulting equation, though intrinsically correct, is not strictly applicable. Moreover, it is not possible to obtain ideal-filter data from real-filter data and the filter characteristic. A complete discussion of this problem is beyond the scope of this report. The fact that the inverse calculation of reference 2 seems to be correct is attributable to the units precision of the results, since the errors are expected to be less than 1/2 decibel for this example.

Data are given next to illustrate the errors involved in not employing an integration procedure to account for air attenuation in band data. The errors shown in figure 5 are introduced by using discrete frequency attenuation at the band center frequency in place

of the attenuation obtained from the present numerical integration method. These examples illustrate the errors that will result in lossless data obtained from air-attenuated data for ambient conditions of 298. 15 K and 70 percent relative humidity. The dependence of the error on frequency and slope of the spectrum as designated by the data slope is shown. Note that these results are illustrative only and that errors of different magnitudes will occur for other ambient conditions and propagation distances. These data illustrate, however, that for short propagation distances, even to extremely high frequencies, the errors tend to be small. On the other hand, there are conditions, especially for longer distances, under which the errors can be significant. These observations are consistent with the conclusions of reference 2.

Figure 6 shows the errors that can result in lossless data by assuming that band attenuation is linearly proportional to distance. Here, too, the errors shown are intended to be illustrative, and they will differ for other conditions.

The significance of these data is that under some conditions approximate methods produce errors that may exceed the precision required for various purposes. Obviously, for suitably conditioned data these errors can be negligible. But to insure small errors requires that data be monitored. The potential for error of any significance can be fully eliminated along with the need for data monitoring by the routine use of numerical integration methods.

CONCLUDING REMARKS

The use of discrete-frequency atmospheric attenuation at some characteristic frequency in a band to represent the band attenuation is inherently erroneous. A method has been developed herein that properly computes fractional-octave band atmospheric attenuation. Ideal filters are assumed, and the total method is reduced basically to two short computer programs. Three logically separable elements are involved: a scheme to infer the spectrum shape from band data, a numerical integration procedure, and discrete-frequency atmospheric attenuation. The first two elements are incorporated in a short Fortran program of which two versions are given - program BASPAT, which is restricted to a homogeneous atmosphere, and program BASLAT, which is essentially the same but permits calculations for a variable atmosphere. This numerical integration program, in both versions, has been written to be understandable and computationally efficient. The discrete-frequency attenuation computational method is essentially taken from the literature and is given as the short program ATMAT. This program may be replaced at the discretion of the reader.

It is shown by example that the effect of the filter is small and that the results computed agree with another published numerical integration method. Examples are also given of errors that occur by not using such methods.

The present method overcomes the shortcomings of other published methods, is inherently simple, and is incorporated in short uncomplicated computer programs. In view of the correctness of the numerical integration procedure and the inherent simplicity of the present method and its implementation, this method is offered as a replacement for discrete-frequency approximate methods where routine computer processing of data is employed.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 1, 1978,
505-04.

APPENDIX - COMPUTER PROGRAMS

Given herein are the Fortran computer programs that constitute the complete method to compute the effect of atmospheric absorption in sound pressure level data comprising the results of fractional-octave band analyses of propagated sound pressure spectra. The program listings contain descriptive information on their functions and use and on the variables employed.

NUMERICAL INTEGRATION

Subroutine BASPAT

Subroutine BASPAT (BAnd SPectrum ATtenuation) uses the scheme previously described to infer the subband sound pressure levels from the input data and performs the calculations specified by equations (10), (11), (17), (18), and (22).

The value of M is coded as 3, which by equation (16), specifies that each band integration is approximated by a sum of seven terms. This has been found by the author to be quite satisfactory in general, but it may be easily changed at the discretion of the reader by changing the value of M. (If M is made greater than 5, it will be necessary to increase the dimension of CL and ~~CM~~.) CR

The variables SL1 and SL2 are the values of the subband levels used in the summations divided by 10. If it is desired not to use the straight line segment scheme to infer the subband levels, it is only necessary to recode as desired the two statements defining these variables (and delete the statements defining the line segment slopes).

Although units are specified in the listing of program BASPAT, the units are, in fact, arbitrary except that the input data are required to be in decibels. The units specified for temperature, humidity, and distance are dictated by the subroutine ATMAT, which is called to obtain discrete-frequency attenuation.

```
      SUBROUTINE BASPAT(SLI,RI,IBNF,NF,NB,T,RH,IDK,RO,SLO,OASPL)
C                /BASPAT - BAND SPECTRUM ATTENUATION/
C  * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C  *                                                                     *
C  *  ADJUSTS FRACTIONAL-OCTAVE BAND SPECTRUM ANALYSIS SOUND PRESSURE    *
C  *  LEVEL DATA FOR THE EFFECT OF ATMOSPHERIC ATTENUATION.  SINGLE-     *
C  *  VALUED BAND ATTENUATIONS ARE OBTAINED BY NUMERICAL INTEGRATION     *
C  *  OF THE DISCRETE-FREQUENCY ATTENUATION EFFECT ON SUBBAND            *
C  *  RELATIVE LEVELS.  SUBBAND RELATIVE LEVELS ARE OBTAINED FROM A      *
C  *  STRAIGHT LINE REPRESENTATION OF THE INPUT DATA AS A FUNCTION OF    *
C  *  THE LOGARITHM OF FREQUENCY.  BAND ATTENUATIONS VARY NONLINEARLY    *
C  *  WITH DISTANCE.  IDEAL FILTERS ARE ASSUMED.  AIR-ATTENUATED         *
C  *  INPUT DATA YIELD LOSSLESS RESULTS;  LOSSLESS INPUT DATA YIELD      *
C  *  AIR-ATTENUATED RESULTS.                                            *
```

```
C  *                                                                     *
C  *  SLI(NF)    INPUT ARRAY OF FRACTIONAL-OCTAVE BAND SOUND PRESSURE    *
C  *             LEVELS, DECIBELS                                        *
C  *  RI         PROPAGATION DISTANCE ASSOCIATED WITH INPUT DATA,        *
C  *             METERS                                                  *
C  *  IBNF       INTERNATIONAL BAND NUMBER OF FIRST FREQUENCY BAND       *
C  *  NF         NUMBER OF FREQUENCY BANDS                               *
C  *  NB         SIZE OF BANDS, 1/NB-OCTAVE                              *
C  *  T          AMBIENT TEMPERATURE, DEGREES KELVIN                     *
C  *  RH         AMBIENT RELATIVE HUMIDITY, PERCENT                      *
C  *  IDK        INPUT DATA KIND INDICATOR                               *
C  *             IDK=0 DESIGNATES AIR-ATTENUATED INPUT DATA,             *
C  *             OTHERWISE LOSSLESS INPUT DATA ASSUMED                   *
C  *  RO         PROPAGATION DISTANCE REQUIRED FOR OUTPUT DATA,          *
C  *             METERS                                                  *
C  *  SLO(NF)    OUTPUT DATA CORRESPONDING TO INPUT DATA ADJUSTED        *
C  *             FOR INVERSE-SQUARE ATTENUATION AND ATMOSPHERIC          *
C  *             ATTENUATION                                             *
C  *  OASPL      OVERALL SOUND PRESSURE LEVEL OF OUTPUT DATA, DECIBELS   *
C  *                                                                     *
C  *  CALLS SUBROUTINE ATMAT                                             *
C  *                                                                     *
C  * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DIMENSION SLI(NF),SLOPE(30,2),CL(5),CR(5),SLO(NF)
      M=3
      SIGN=-0.1
      IF(IDK.EQ.0)SIGN=+0.1
      R=RO
      IF(IDK.EQ.0)R=RI
      RSQ=20.0*ALOG10(RI/RO)
      CB=0.3/FLOAT(NB)
      CN=2*M+1
      NLS=NF-1
      DO 1 I=1,NLS
      SLOPE(I,2)=(SLI(I+1)-SLI(I))/(10.0*CN)
    1 SLOPE(I+1,1)=SLOPE(I,2)
      SLOPE(1,1)=SLOPE(1,2)
      SLOPE(NF,2)=SLOPE(NF,1)
      DO 2 J=1,M
      CL(J)=10.0**(CB/CN*FLOAT(-J))
    2 CR(J)=10.0**(CB/CN*FLOAT(J))
      OASPL=0.0
      DO 4 I=1,NF
      FC=10.0**(CB*FLOAT(I+IBNF-1))
      CALL ATMAT(T,RH,R,FC,AC)
      SUMN=10.0**(SIGN*AC)
      SUMD=1.0
      DO 3 J=1,M
      F1=FC*CL(J)
      F2=FC*CR(J)
      SL1=SLOPE(I,1)*FLOAT(-J)
      SL2=SLOPE(I,2)*FLOAT(J)
      CALL ATMAT(T,RH,R,F1,A1)
      CALL ATMAT(T,RH,R,F2,A2)
      SUMN=SUMN+10.0**(SL1+SIGN*A1)+10.0**(SL2+SIGN*A2)
    3 SUMD=SUMD+10.0**SL1+10.0**SL2
      SLO(I)=SLI(I)+10.0*ALOG10(SUMN/SUMD)+RSQ
```

```
    4 OASPL=OASPL+10.0**(SLO(I)/10.0)
      OASPL=10.0*ALOG10(OASPL)
      RETURN
      END
```

Subroutine BASLAT

Subroutine BASLAT (BAnd Spectrum Layered ATtenuation) is an alternate version of BASPAT that is expanded to allow calculations for a layered atmosphere. The comments in the text of the preceding section apply to BASLAT as well.

It should be noted that BASLAT differs only in that it allows the discrete-frequency attenuation along the path to be obtained as the sum of incremental attenuations. Another way to accomplish the same result is to use BASPAT as written and to modify the program for discrete-frequency attenuation (ATMAT) so that it returns a result obtained by summation along the path.

```
      SUBROUTINE BASLAT(SLI,IDK,IBNF,NF,NB,NL,T,RH,RINC,RL,SLO,OASPL)
C                /BASLAT - BAND SPECTRUM LAYERED ATTENUATION/
C  * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C  *                                                                     *
C  *  THIS IS AN ALTERNATE VERSION OF BASPAT THAT HANDLES INCREMENT-     *
C  *  ALLY VARIABLE ATMOSPHERIC PROPERTIES ALONG THE PROPAGATION         *
C  *  PATH, WHICH MIGHT BE REQUIRED BY A LAYERED ATMOSPHERE MODEL,       *
C  *  FOR EXAMPLE.  IF A SINGLE INCREMENT (LAYER) IS SPECIFIED           *
C  *  (I.E., NL=1), THIS PROGRAM PERFORMS EXACTLY AS BASPAT.             *
C  *                                                                     *
C  *  SLI(NF)    INPUT ARRAY OF FRACTIONAL-OCTAVE BAND SOUND PRESSURE    *
C  *             LEVELS, DECIBELS                                        *
C  *  IDK        INPUT DATA KIND INDICATOR                               *
C  *             IDK=0 DESIGNATES AIR-ATTENUATED INPUT DATA,             *
C  *             OTHERWISE LOSSLESS INPUT DATA ASSUMED                   *
C  *  IBNF       INTERNATIONAL BAND NUMBER OF FIRST FREQUENCY BAND       *
C  *  NF         NUMBER OF FREQUENCY BANDS                               *
C  *  NB         SIZE OF BANDS, 1/NB-OCTAVE                              *
C  *  NL         NUMBER OF PATH LENGTH INCREMENTS; FOR A HOMOGENEOUS     *
C  *             ATMOSPHERE, SET NL=1                                    *
C  *  T(NL)      AIR TEMPERATURE IN EACH PATH INCREMENT, DEGREES         *
C  *             KELVIN                                                  *
C  *  RH(NL)     RELATIVE HUMIDITY IN EACH PATH INCREMENT, PERCENT       *
C  *  RINC(NL)   PROPAGATION PATH LENGTH INCREMENTS FOR THE AIR-         *
C  *             ATTENUATED DATA, METERS                                 *
C  *  RL         PROPAGATION DISTANCE IN METERS FOR THE LOSSLESS DATA    *
C  *  SLO(NF)    OUTPUT DATA CORRESPONDING TO INPUT DATA ADJUSTED        *
C  *             FOR INVERSE-SQUARE ATTENUATION AND ATMOSPHERIC          *
C  *             ATTENUATION                                             *
C  *  OASPL      OVERALL SOUND PRESSURE LEVEL OF OUTPUT DATA, DECIBELS   *
C  *                                                                     *
C  *  CALLS SUBROUTINE ATMAT                                             *
C  *                                                                     *
C  * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DIMENSION SLI(NF),SLOPE(30,2),CL(5),CR(5),SLO(NF)
      DIMENSION T(NL),RH(NL),RINC(NL)
      M=3
```

```
      SIGN=-0.1
      IF(IDK.EQ.0)SIGN=+0.1
      RA=0.0
      DO 1 K=1,NL
    1 RA=RA+RINC(K)
      RSQ=(10.0*SIGN)*20.0*ALOG10(RA/RL)
      CB=0.3/FLOAT(NB)
      CN=2*M+1
      NLS=NF-1
      DO 2 I=1,NLS
      SLOPE(I,2)=(SLI(I+1)-SLI(I))/(10.0*CN)
    2 SLOPE(I+1,1)=SLOPE(I,2)
      SLOPE(1,1)=SLOPE(1,2)
      SLOPE(NF,2)=SLOPE(NF,1)
      DO 3 J=1,M
      CL(J)=10.0**(CB/CN*FLOAT(-J))
    3 CR(J)=10.0**(CB/CN*FLOAT(J))
      OASPL=0.0
      DO 7 I=1,NF
      FC=10.0**(CB*FLOAT(I+IBNF-1))
      AC=0.0
      DO 4 K=1,NL
      CALL ATMAT(T(K),RH(K),RINC(K),FC,DELTA)
    4 AC=AC+DELTA
      SUMN=10.0**(SIGN*AC)
      SUMD=1.0
      DO 6 J=1,M
      F1=FC*CL(J)
      F2=FC*CR(J)
      SL1=SLOPE(I,1)*FLOAT(-J)
      SL2=SLOPE(I,2)*FLOAT(J)
      A1=0.0
      A2=0.0
      DO 5 K=1,NL
      CALL ATMAT(T(K),RH(K),RINC(K),F1,DELTA1)
      CALL ATMAT(T(K),RH(K),RINC(K),F2,DELTA2)
      A1=A1+DELTA1
    5 A2=A2+DELTA2
      SUMN=SUMN+10.0**(SL1+SIGN*A1)+10.0**(SL2+SIGN*A2)
    6 SUMD=SUMD+10.0**SL1+10.0**SL2
      SLO(I)=SLI(I)+10.0*ALOG10(SUMN/SUMD)+RSQ
    7 OASPL=OASPL+ 10.0**(SLO(I)/ 10.0)10.0
      OASPL=10.0*ALOG10(OASPL)
      RETURN
      END
```

Subroutine ATMAT

Subroutine ATMAT (for ATmospheric ATtenuation) is called by BASPAT and BASLAT to obtain total-path discrete-frequency atmospheric attenuation for a homogeneous atmosphere. The substance of this program is taken from reference 2.

```
      SUBROUTINE ATMAT (T,RH,DIST,FREQ,ATT)
C                 /ATMAT - ATMOSPHERIC ATTENUATION/
C  * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C  *                                                                       *
C  *  COMPUTES ATMOSPHERIC ATTENUATION IN DECIBELS FOR SPECIFIED           *
C  *  TEMPERATURE, RELATIVE HUMIDITY, DISTANCE, AND FREQUENCY              *
C  *  FOLLOWING BASS AND SHIELDS, NASA CR-2760, 1977.                      *
C  *  APPLICABLE TO FREQUENCIES FROM 50 TO 100000 HERTZ, TEMPERATURES      *
C  *  FROM 255.4 TO 310.9 DEGREES KELVIN, RELATIVE HUMIDITIES FROM         *
C  *  ZERO TO ONE HUNDRED PERCENT.                                         *
C  *                                                                       *
C  *  T          TEMPERATURE, DEGREES KELVIN                               *
C  *  RH         RELATIVE HUMIDITY, PERCENT                                *
C  *  DIST       DISTANCE, METERS                                          *
C  *  FREQ       FREQUENCY, HERTZ                                          *
C  *  ATT        ATTENUATION, DECIBELS                                     *
C  *                                                                       *
C  *  P          AMBIENT PRESSURE IN ATMOSPHERES, MAY BE MADE A            *
C  *             CALLING VARIABLE (SEE CR-2760)                            *
C  *                                                                       *
C  * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      P=1.0
      T1=T/293.
      T01=273.16
      PS=10.79586*(1.0-T01/T)-5.02808*ALOG10(T/T01)                      -
     1+1.50474E-4*(1.0-10.0**(-8.29692*((T/T01)-1.0)))                   -
     2+0.42873E-3*(10.0**(4.76955*(1.0-(T01/T)))-1.0)-2.2195983
      PS=10.0**PS
      H=RH*PS/P
      FRO=P*(24.0+4.41E+4*H*(0.05+H)/(0.391+H))
      FRN=P/SQRT(T1)*(9.0+350.0*H*EXP(-6.142*((1.0/T1)**0.333-1.0)))
      ALPHA=SQRT(T1)*FREQ**2/P*(1.84E-11+2.1913E-4/T1*P*(2239.1/T)**2    -
     1*EXP(-2239.1/T)/(FRO+(FREQ**2/FRO))+8.1619E-4/T1*P*(3352.0/T)**2   -
     2*EXP(-3352.0/T)/(FRN+(FREQ**2/FRN)))*8.686
      ATT=ALPHA*DIST
      RETURN
      END
```

REFERENCES

1. Standard Values of Atmospheric Absorption as a Function of Temperature and Humidity. Aerospace Recommended Practice 866A, revised March 15, 1975, SAE.
2. Bass, H. E.; and Shields, F. D.: Atmospheric Absorption of High Frequency Noise and Application to Fractional-Octave Bands. NASA CR-2760, 1977.
3. Marsh, Alan H.: Atmospheric-Absorption Adjustment Procedure for Aircraft Flyover Noise Measurements. DYTEC-R-7705, Dytec Engineering, Inc., 1977. (FAA-RD-77-167, AD-A051700.)
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5. Standard Values of Atmospheric Absorption as a Function of Temperature and Humidity. Aerospace Recommended Practice 866, August 31, 1964, SAE.
6. Preferred Frequencies and Band Numbers for Acoustical Measurements. Am. Natl. Stands. Inst. S1.6-1967 (R1976).
7. Octave, Half-Octave, and Third-Octave Band Filter Sets. Am. Natl. Stands. Inst. S1.11-1966 (R1975).

TABLE I. - COMPARISON OF NUMERICAL INTEGRATION RESULTS

FROM PRESENT METHOD WITH THOSE OF REFERENCE 2

International band number	1/3-Octave band nominal center frequency, Hz	1/3-Octave band sound pressure levels at 20 meters from source				
		Original lossless	Air-attenuated at 293 K, 50 percent relative humidity		Inverse calculation to lossless	
			Present method	Reference 2	Present method	Reference 2
36	4 000	40.0	39.5	-----	40.0	--
37	5 000	37.0	36.2	36.2	37.0	--
38	6 300	34.0	32.8	32.8	34.0	34
39	8 000	31.0	29.1	29.2	31.0	31
40	10 000	28.0	25.1	25.2	28.0	28
41	12 500	25.0	20.6	20.8	25.0	25
42	16 000	22.0	15.4	15.5	22.0	22
43	20 000	19.0	9.3	9.5	19.0	19
44	25 000	16.0	1.9	2.2	16.0	16
45	31 500	20.0	.1	.4	20.0	20
46	40 000	24.0	-2.5	-2.4	24.0	24
47	50 000	28.0	-6.1	-5.6	28.0	28
48	63 000	32.0	-10.9	-10.2	32.0	32
49	80 000	36.0	-17.5	-17.1	36.0	--
50	100 000	40.0	-27.3	-----	41.0	--

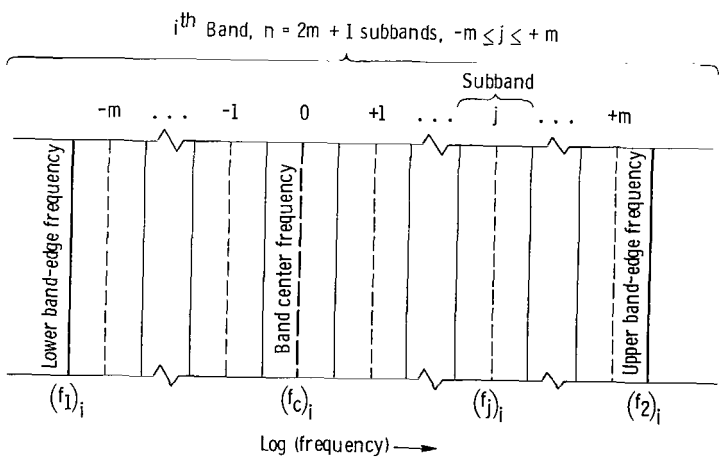


Figure 1. - Delineation of fractional-octave bands and subbands on logarithmic frequency scale.

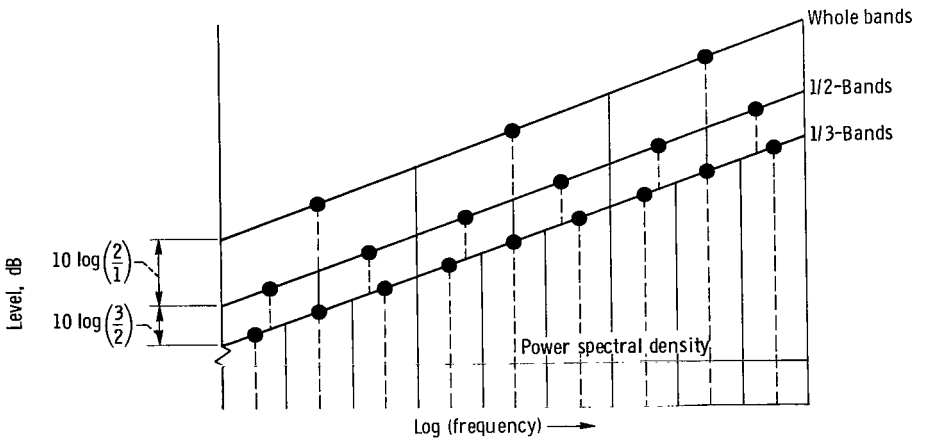


Figure 2. - Relative slope and amplitude relationships between band levels and power spectral density.

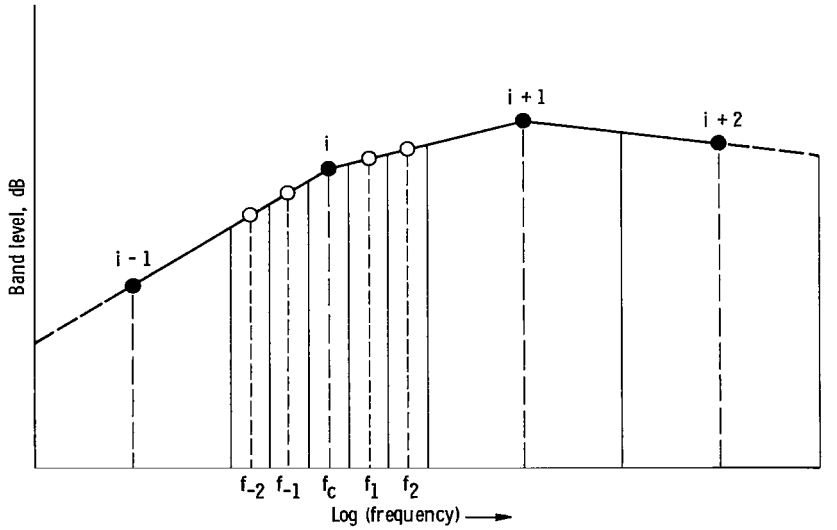


Figure 3. - Scheme to infer subband level distribution across each band of fractional-octave band analysis using straight-line half-segments. Solid symbols denote band levels; open symbols denote subband levels; m = 2, n = 2m + 1 = 5, for this example.

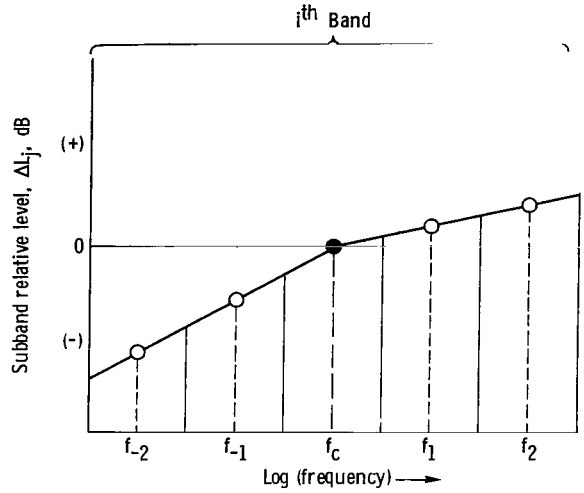
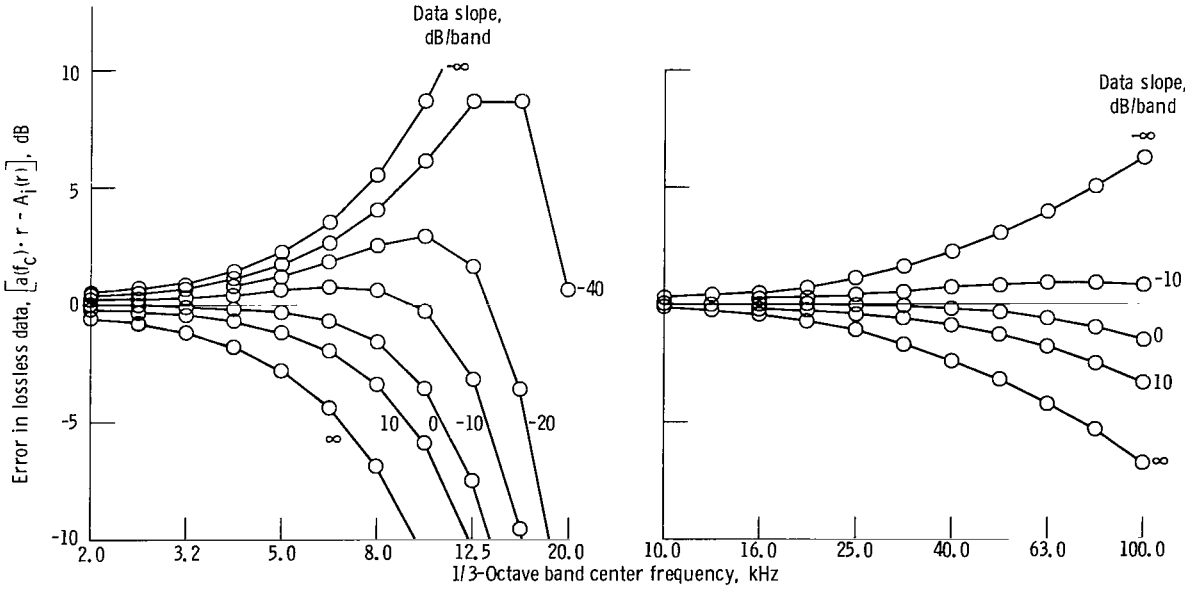


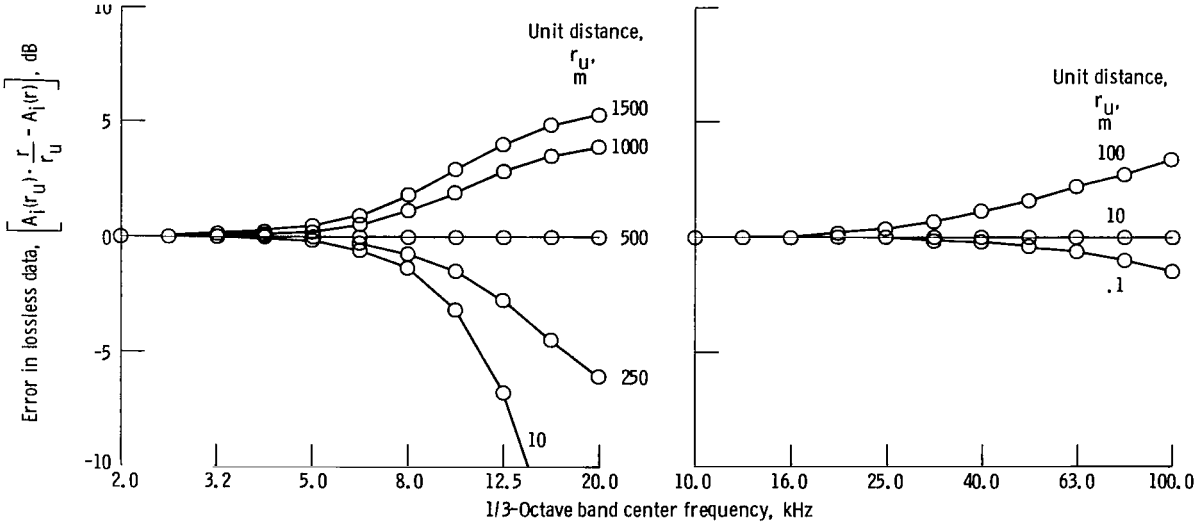
Figure 4. - Subband level distribution relative to subband level at band center frequency.



(a) Frequency range, 2 to 20 kilohertz for 500-meter propagation distance.

(b) Frequency range, 10 to 100 kilohertz for 10-meter propagation distance.

Figure 5. - Error in lossless 1/3-octave band data obtained from air-attenuated data at 298.15 K, 70 percent relative humidity by using center frequency attenuation $a(f_c) \cdot r$ instead of integrated results from present method $A_i(r)$.



(a) Frequency range, 2 to 20 kilohertz for 500-meter propagation distance.

(b) Frequency range, 10 to 100 kilohertz for 10-meter propagation distance.

Figure 6. - Error in lossless 1/3-octave band data obtained from air-attenuated data at 298.15 K, 70 percent relative humidity by assuming band attenuation is proportional to distance. Attenuation for each unit distance obtained by numerical integration and proportioned to actual propagation distance to obtain approximate results $A_i(r_u)\cdot r/r_u$. Resulting differences from integrated results for actual distance $A_i(r)$ are shown.

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15. Supplementary Notes

16. Abstract

The atmosphere is acoustically dissipative and causes significant attenuation of propagating sound in addition to the attenuation caused by spherical spreading. Since the effect is frequency dependent, resulting in observed spectra that are not representative of the source, it is important to account for it in much acoustical work. Commonly used acoustical data are the results of fractional-octave band analyses of spectra, whereas atmospheric attenuation is known fundamentally for discrete-frequency sounds. Correct methods of accounting for atmospheric attenuation in band data require consideration of the integrated effect across the bands for the specific distance involved. Such methods have not found their way into general use. Published analyses and computer programs have problems that inhibit implementation of correct methods by potential users. This report illustrates the underlying simplicity of an integral approach, documents the basic analysis, and gives examples of errors incurred by not using correct methods. Computer programs are provided that are understandable, efficient, and simple to use. It is hoped that this will facilitate more widespread use of correct computational methods, especially where routine computer processing of data is employed.

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